Efficient analysis of phase-locked loops through a novel time-frequency approach, based on two envelope transient formulations

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ABSTRACT — **A novel time-frequency analysis for phase-locked loops (PLL) is presented here, based on two nested envelope transient formulations. The outer envelope is centered, as usual, about the VCO frequency. The inner-envelope is obtained when considering a frequency domain expansion of the loop state variables about the reference frequency and its multiples, with slow-varying phasors. Thus, the influence of these frequency components on the loop behavior is taken into account. The double envelope approach enables an efficient and realistic simulation of the acquisition times in the common case of slow time response of the loop. On the other hand, the phase-noise can be analyzed from very low offsets from the carrier, while taking into account the spurious content due to the reference frequency. In the absence of noise perturbations, the application of harmonic balance to the VCO envelope enables an accurate and efficient analysis of the PLL versus any parameter of interest. The combination with continuation techniques to circumvent the possible turning points is straightforward. The stability is analyzed through the calculation of the eigenvalues of the harmonic-balance characteristic matrix. Very good agreement has been obtained in the comparison with time domain simulation and experimental results.**

I. INTRODUCTION

The nonlinear analysis of phase-locked loops (PLL) is usually based on an envelope formulation about the VCO output frequency [1-2]. Realistic models of the phase detector give rise to spurious frequencies coming from the harmonics of the reference oscillator. These terms are responsible for the incidental frequency modulation and may have big influence on the spectrum and even on the stability of the steady state solution [2]. The integration time rate of the VCO envelope equations is determined by the frequency of this reference oscillator. Then, for the case of a low cut-off frequency of the loop filter, the transients can turn out to be extremely long, due to the small bandwidth of the loop. An enormous amount of simulation points will be required and, in these conditions, reaching the steady state or even deciding whether the loop is in locked or unlocked may be a difficult matter. Here a harmonic balance technique is proposed to solve the VCO envelope equations in an efficient manner, while taking into account all the spurious content due to the reference oscillator. The loop variables are expanded in a Fourier series with the reference frequency as fundamental. This kind of resolution will have all the advantages of the harmonic balance method, i.e., fast convergence, compared to time-domain integration and straightforward application of continuation techniques.

On the other hand, when spurious from the reference oscillator are present in the loop, the phase noise analysis at low frequency offsets from the VCO carrier can be virtually impossible, due the high value of the reference frequency, in the order of the ten of MHz, compared to the noise frequency. The harmonic balance formulation at the reference frequency can overcome this problem by considering the phasors of the frequency expansions of the loop state variables as time varying. Thus the loop equations are formulated as two nested envelopes. The outer envelope is centered, as usual, about the VCO frequency. The inner envelope is centered about the reference frequency.

The two new techniques have been applied to a microwave PLL in the 2-3 GHz band, comparing the results with time-domain integration and measurements.

II. HARMONIC BALANCE ANALYSIS

a) Harmonic-balance formulation

The nonlinear equations of the phase-locked loop are usually expressed in terms of the voltage of the reference oscillator $v_i(t)$ and that of the VCO output $v_o(t)$. In the simplified model, for which the incidental FM is not taken into account, the phase-locked solutions are given by a constant phase shift ϕ_o between the reference oscillator and the frequency divider output signal. When the incidental FM is taken into account, the steady-state phase shift varies in time $\phi_o(t)$, due to the presence of spurious frequencies from the reference oscillator. The VCO output voltage is expressed as an analytic signal: $\mathrm{Re}\left\{V_o e^{j\theta_0(t)} e^{j2\pi f_0 t}\right\}$, where the VCO phase θ_o is a possibly nonlinear function of the input voltage: $\theta_o = f(v)$. In the new analysis technique, the loop variables are expanded in a Fourier series at the reference frequency ω_i. Then the voltage at the output of the frequency divider is:

$$v_d(t) = \mathrm{Re}\left\{K_{div} V_o e^{j\left(\frac{f(v)}{N} + \frac{2\pi f_o t}{N}\right)}\right\} = \sum_k V_{dk} e^{jk\omega_i t} \quad (1)$$

where a frequency expansion in terms of ω_i has been carrier out. The voltage of the reference oscillator can be expressed: $v_i(t) = V_i/2e^{j\omega_i t} + V_i/2e^{-j\omega_i t}$. Then the harmonic terms of $v_d(t)$ and v(t) are related through the equations:

$$\bar{I} = \bar{H}_{PD}(\bar{V}_i, \bar{V}_d) \quad (2)$$

$$A(\omega_o)\bar{V} = B(\omega_o)\bar{I} \quad (3)$$

where H_{PD} is the harmonic equation associated to the nonlinear model of the phase detector, and (3) is the loop filter transfer function. The vectors I, V_i, V_d and V contain the harmonic components $k\omega_i$ of the respective loop variables. The equations (1) to (3) compose the harmonic balance system to be solved through the Newton-Raphson algorithm. The resolution time, contrarily to time domain integration, is not affected by the loop bandwidth.

The above analysis has been applied to a type II third-order phase-locked loop in the microwave band 2-3 GHz. Both a JK and a frequency mixer phase detector have been used for the analysis. To enable the introduction of the digital phase detector in the HB system, the frequency components at the output of this detector are analytically obtained from the duty cycle of the pulsed signal at the detector output. Since it is a phase-frequency detector, this duty cycle depends both on the phase of the reference signal and the phase of the feedback signal. The results of the new harmonic-balance technique when a JK phase detector is used are compared in Fig. 1 with those from time-domain integration with perfect agreement. The waveform corresponds to one period of the VCO input signal at the reference frequency.

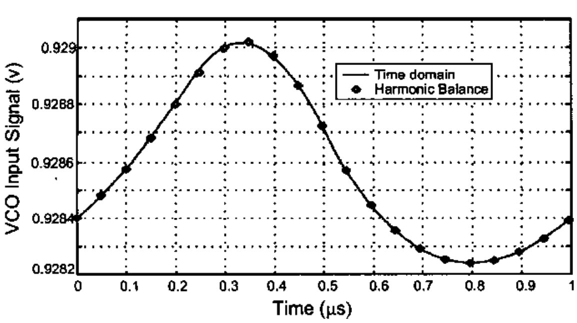

Fig. 1 Comparison between time and frequency domain simulations of the PLL with a digital phase detector. The waveform corresponds to the VCO input, which introduces reference spurious components at the VCO output spectrum.

The DC component fixes the VCO output center frequency to 2,165 GHz, while the oscillating components introduce spurious at the reference frequency f_i = 1MHz.

b) Parametric analysis

It is usually desirable to predict the evolution of the PLL solution in terms of parameters such as the loop gain or the division order. To do so, the parameter is introduced into the harmonic balance system (1) to (3). In order to obtain the curve for which the PLL is in phase-locked behavior, an original technique is employed here. The average of the phase shift $<\phi_o(t)>$ is swept between 0 and 2π, calculating, for each phase, the parameter value and the harmonic components of the loop variables through equations (1) to (3).

In the case of a mixer phase detector, the continuation technique provides, for three different values of one of the filter poles, the closed curves of Fig. 2a. The ellipsoidal curves are not symmetric due to the influence of frequency characteristic of the loop filter. The upper section of each curve corresponds to stable periodic solutions or nodes, as has been verified through the stability analysis technique in c). The lower section corresponds to unstable periodic solutions or saddles [3]. The synchronization band is delimited by two local/global saddle node bifurcations, at which an invariant cycle is formed in the Poincaré map, giving rise to the unlocked quasi-periodic solution [3]. Note that, as the loop-filter pole approaches the origin, this band increases. It would become infinite for a filter pole located at the origin, which is an ideal situation. In Fig. 2a, the amplitude values remain in the same order, since the rest of filter parameters have been kept unchanged for better bandwidth comparison. Fig. 2b presents the variation of the spurious of the reference-frequency (in dBc) at the VCO output versus the loop gain. The maximum power level of this spurious if often specified as a design constraint and the new technique enables a straightforward prediction of its variation versus any parameter, through continuation techniques.

Aside from the substantial reduction of computing time and memory, one essential advantage of the HB technique is the elimination of the uncertainty about the locked or unlocked state of the loop. Actually, close to the border of the synchronization band, the duration of the transient increases, due to the proximity to the bifurcation [2]. When the system is unlocked, an intermittency phenomenon is observed (associated to the saddle-node bifurcations), which gives rise to an apparently periodic waveform for long time intervals. The practical limitations of the simulation time, in the time domain integration, may lead to the erroneous conclusion that the system is in locked state. The problem of the long duration of the transients is even more serious when one of the loop poles is close to the origin. Actually, the system always operates in a near-bifurcation state, with the bifurcation being actually obtained for the pole at the origin. The intermittency phenomenon is illustrated in Fig. 3, for an N value outside the phase-locked band predicted by the HB analysis of Fig. 2a.

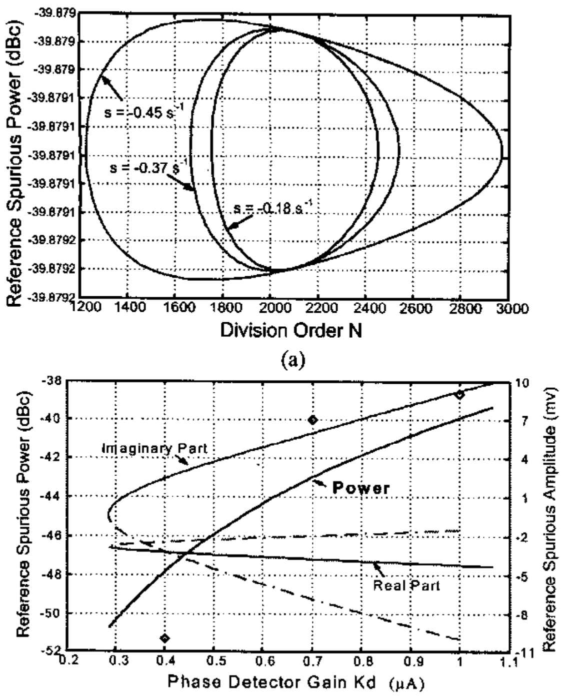

Fig. 2 Continuation technique. (a) Synchronization curves versus the frequency division order N, for different values of one of the loop-filter poles. (b) Evolution of the reference-frequency spurious at the VCO output versus the phase-detector gain. Measurements are superimposed.

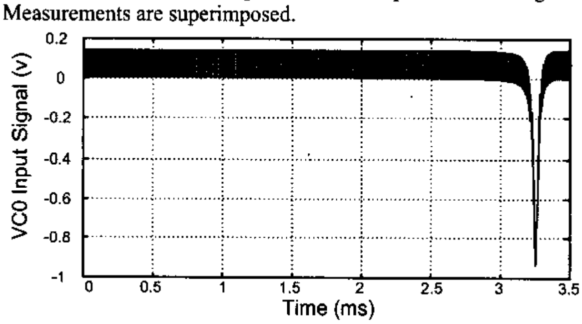

Fig. 3 Intermittency phenomenon, after the loss of synchronization. It can mislead about the locked or unlocked state of the PLL.

c) Stability analysis

The harmonic balance formulation (1) to (3) enables a simple stability analysis of the phase-locked states. This is done by considering a perturbation of the form $e^{\sigma + j\omega}$ and linearizing (1) to (3) about the nonlinear periodic solution at ω_i. The reduced dimension of the system allows the numerical calculation of the eigenvalues $(\sigma_k + j\omega_k)$ of the resulting characteristic system. The Floquet multipliers, enabling a detailed prediction of the solution reaction to perturbations, are obtained through $\mu_k = e^{j(\sigma_k + j\omega_k)T}$, with T being the period of the reference oscillator. In a former work [2], these multipliers were calculated by linearizing the nonlinear differential equations about the periodic solution and integrating from independent initial vectors of state variables. In comparison, the new technique is more efficient and substantially reduces the computational cost. The combination with the HB continuation method enables obtaining the variation of the multipliers along the entire solution curves, including unstable sections. In this case, the whole synchronization band, limited by the saddle-node bifurcations, is stable.

III. DOUBLE ENVELOPE ANALYSIS

For the double envelope analysis of the phase locked loop, the phasors in the system (1) to (3) are considered as time varying [4-6]. Then the following system of differential equations in the state-variable phasors is obtained:

$$v_d(t) = \sum_k V_{dk}(t)\, e^{jk\omega_i t}$$
$$v_i(t) = \frac{V_i(t)}{2} e^{j\omega_i t} + \frac{V_i(t)}{2} e^{-j\omega_i t} \qquad (4)$$
$$\bar{I}(t) = \bar{H}_{PD}(\bar{V}_i(t), \bar{V}_d(t))$$

$$A(\omega_o + D_t)\bar{V} = B(\omega_o + D_t)\bar{I}$$

where $D_t = d/dt$ is the time derivative operator. Note that the formulation above employed does not rely on a narrowband assumption, since no limitation is imposed in the order of time derivatives contained in the last equation. It can be applied regardless of the bandwidth of the envelopes about each frequency component $k\omega_i$, in similar way to the circuit formulations given in [5-6].

The double-envelope analysis enables the simulation of the PLL acquisition times. This analysis is specially useful for loops with a narrow bandwidth, which is a common case in designs with strict phase-noise specifications. It also allows the simulation of the unlocked states or rotations [2], which can be essential for the prediction of the usually observed hysteresis phenomena. Fig. 4 shows the application of the double envelope for the same parameter values as in Fig. 3, beyond the hold-in range. It shows the same intermittency phenomenon that had already been observed in the time-domain simulation, with a substantial reduction of the computation time, since the integration time step is about 10ns and the envelope time step is about 20μs.

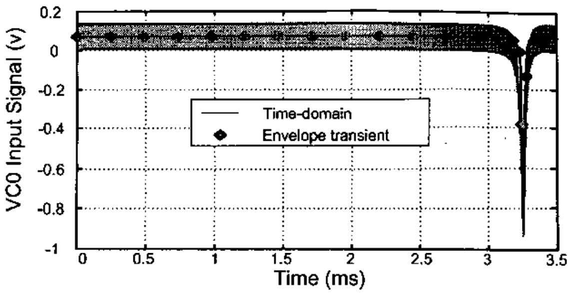

Fig. 4 Double envelope analysis of the intermittency after the loss of synchronization.

The equations (4) can be applied for the noise analysis of the phase-locked loop, with noise contributions from the reference oscillator, the VCO or the frequency divider. These contributions are described here as a summation of pseudo sinusoids, as was done in [4]. The JK phase detector is used for this analysis. For low-amplitude noise, the non-autonomous system (4) can be linearized about the nonlinear steady state solution with a substantial decrease of computing time. The resulting spectrum is shown in Fig. 5.

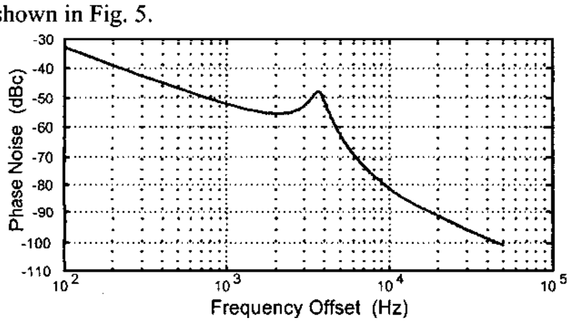

Fig. 5 Double envelope calculation of the phase-noise spectrum at the VCO output, when employing a digital phase detector.

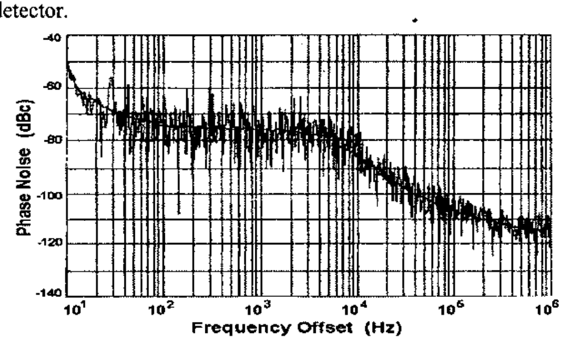

Fig. 6 Experimental phase-noise spectrum

As can be seen, the envelope transient approach is able to predict the noise shoulder due to insufficient stability margin [1-2]. In contrast with former techniques, now the calculation can be carried out for arbitrarily small offset frequency from the carrier. The simulated spectrum can be compared with the experimental one in Fig. 6.

V. CONCLUSION

A new technique has been presented for efficiently analyzing phase-locked loops in the presence of the spurious frequency components due to the reference oscillator. The noise-free loop is solved through harmonic balance at the reference frequency, which enables a fast resolution of systems with long transient responses and the application of continuation techniques to obtain the phase-locked curves versus any parameter of interest. The stability of the solution is calculated through an eigenvalue resolution of the perturbed harmonic balance system. For an accurate and efficient noise analysis, the loop equations are formulated as two nested envelopes, respectively centered about the VCO frequency and the reference frequency. The new analysis has been applied to a microwave phase-locked loop, operating in the 2-3 GHz band. Excellent results have been obtained in the comparison with standard time domain integration and in the comparison with measurements.

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